Similar Solutions of Boundary-Layer Equations for Power-Law Fluids

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The authors have recently obtained (*1*) the condition under which the boundary-layer equations for two-dimensional flows of power-law fluids (*2*) admit of similar solutions. Schowalter (*3*) used a somewhat different method for the three-dimensional case of the flow past a flat plate where the potential velocity vector is not perpendicular to the leading edge of the plate.

The object of the present note is to point out a small error in the mathematical development of (*3*) and to give the necessary corrections.

We use the notation of (3). The corrected second term on the L. H. S. of Equations (14), (15), and (16) should be

$$\frac{g^{n+1}W^o}{(U^o)^n}\frac{\partial U^o}{\partial z^o}[F'G'-1] \qquad (1)$$

$$\frac{W^o}{(U^o)^{n-1}}\frac{\partial \ln U^o}{\partial x^o}[F'G'-1] \qquad (2)$$

and

$$\frac{U^o}{(W^o)^{n-1}}\frac{\partial \ln W^o}{\partial x^o}[F'G'-1] \qquad (3)$$

The modified Equation (17) is

$$(U^o)^{1-n}\frac{\partial U^o}{\partial x^o} = a_1 W^o (U^o)^{1-n} \frac{\partial \ln U^o}{\partial z^o} - a_2 (U^o)^{1-n}\frac{\partial W^o}{\partial z^o} =$$

$$a_3 (U^o)^{2-n}\frac{\partial \ln g}{\partial x^o} = a_4 (U^o)^{1-n}$$

$$W^o\frac{\partial \ln g}{\partial z^o} = \frac{a_5}{g^{n+1}} = a_6 (W^o)^{1-n}$$

$$\frac{\partial W^o}{\partial z^o} = a_7 U^o (W^o)^{1-n}\frac{\partial \ln W^o}{\partial x^o} =$$

$$a_8 (W^o)^{1-n}\frac{\partial U^o}{\partial x^o} = a_9 (W^o)^{2-n}\frac{\partial \ln g}{\partial z^o}$$

$$= a_{10} (W^o)^{1-n} U^o \frac{\partial \ln g}{\partial x^o} \qquad (4)$$

From Equation (4) we get

$$W^o = k\,U^o \qquad (5)$$

and

$$(U^o)^{1-n}\frac{\partial U^o}{\partial x^o} = a_1 k\,(U^o)^{1-n}\frac{\partial U^o}{\partial z^o} =$$

$$a_2 k\,(U^o)^{1-n}\frac{\partial U^o}{\partial z^o} = a_3\frac{(U^o)^{2-n}}{g}\frac{\partial g}{\partial x^o}$$

$$= a_4 k\,(U^o)^{2-n}\frac{1}{g}\frac{\partial g}{\partial z^o} = \frac{a_5}{g^{n+1}} =$$

$$a_6 (kU^o)^{1-n}\frac{\partial U^o}{\partial z^o} = a_7 (kU^o)^{1-n}$$

$$\frac{\partial U^o}{\partial x^o} = a_8 (kU^o)^{1-n}\frac{\partial U^o}{\partial x^o} =$$

$$a_9 (kU^o)^{2-n}\frac{1}{g}\cdot\frac{\partial g}{\partial z^o} =$$

$$a_{10} k^{1-n} (U^o)^{2-n}\cdot\frac{1}{g}\frac{\partial g}{\partial x^o} \qquad (6)$$

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Impact Tube Size in Fluid Velocity Measurement

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The use of impact tubes for fluid velocity measurement requires that four corrections on the raw data be made. The first correction is for velocity gradient (*1*). When the velocity profile is steep, as it is near a wall, it is necessary to take the size of the impact opening into account. The second adjustment is for fluid viscosity (*2*) and is particularly important for highly viscous fluids. Both of these corrections are significant near the wall and become negligible in the turbulent core. The third correction is due to turbulent fluctuations (*4*) and is negligible near the wall, but must be taken into account in the turbulent core region.

The fourth correction is for pressure variation over the impact area and is important in all flow regions. It is this correction to which we wish to confine our attention at the present time. The correction arises because of the nature of the flow around the entire impact tube, and the adjustment is greater for straight impact tubes than for Pitot or bent impact tubes.

In the past, investigators (*3 to 8*) using impact tubes for velocity measurements have reported that their data fall below that obtained by other techniques. Rosler and Bankoff (*4*) have concluded that velocity measurements for submerged water jets using impact tubes give results which are about 12% below those obtained using a hot-wire anemometer. Rothfus and co-workers (*5, 6*), working on flow in conduits, have used several sizes of impact tubes and extrapolated to zero diameter. This method accounts for all but the viscous and turbulent fluctuation corrections.

Consider a fluid flowing steadily past a circular cylinder with its longitudinal axis at right angles to the direction of flow. The pressure distribution on the forward surface of the cylinder is given

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where k is a constant and

$$\left.\begin{array}{l} 1 = a_8\, k^{1-n} = a_7\, k^{1-n}; \\ \qquad a_1 = a_2 = a_6\, k^{1-n} \\ a_3 = a_{10}\, k^{1-n};\ a_4 = a_9\, k^{1-n} \end{array}\right\} \quad (7)$$

We also get

$$\frac{\partial \ln U^o}{\partial x^o} = a_1 k \frac{\partial \ln U^o}{\partial z^o} = a_3 \frac{\partial \ln g}{\partial x^o} = a_4 k \frac{\partial \ln g}{\partial z^o} = a_5 \frac{U^{o\,n-2}}{g^{n+1}} \quad (8)$$

Integrating one obtains

$$U^o = f\,(x^o + Az^o) \quad (9)$$

$$g = [f(x^o + Az^o)]^p \quad (10)$$

where

$$1 = a_1 kA = a_3 p = a_4 kAp \quad (11)$$

and

$$f' = a_5\, f^{(n-1)-p(n+1)} \quad (12)$$

Here A and p are arbitrary constants and f is an arbitrary function.

Integrating Equation (12) and using Equation (5)

$$U^o = c\,(x^o + Az^o)^m \quad (13)$$

$$W^o = c\,k\,(x^o + Az^o)^m \quad (14)$$

$$g = c^p\,(x^o + Az^o)^{pm} \quad (15)$$

where

$$m = \frac{1}{p(n+1) - (n-1)} \quad (16)$$

C is a constant and $p(n+1)-(n-2) \neq 0$ If $p(n+1)-(n-2) = 0$ one obtains the solution

$$U^o = C\, e^{B(x^o + Az^o)} \quad (17)$$

$$W^o = k\,C\, e^{B(x^o + Az^o)} \quad (18)$$

$$g = C'\, e^{B(x^o + Az^o)[(n-2)/(n+1)]} \quad (19)$$

where k, B, and C' are arbitrary constants; Equations (7) and (11) then would determine a_1 to a_{10}.

From Equation (5), it is evident that the potential velocity is in a fixed direction, and from Equations (13) and (14) one finds that similar solutions are possible when the potential velocity is proportional to some power of the distance from a fixed stagnation straight line.

By putting $k = 0$, $A = 0$, one obtains the two-dimensional case.

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LITERATURE CITED

1. Kapur, J. N., and R. C. Srivastava, *Z. Angew. Math. Phys.*, **14**, 4 (1963).
2. Kapur, J. N., *J. Phys. Soc. Japan*, **18**, 144 (1963).
3. Schowalter, W. R., *A.I.Ch.E. Journal*, **6**, 24 (1960).